Topological Relations between Temporal Granular Terms

Throughout the following discussion, the generalization of temporal granular terms occurs between granularities related by the finer than relationship. The generalization of temporal granular terms may affect the temporal topological relationships held between pairs of atoms. On one hand, the type of relationship may change. For instance, we might have a relation between two time intervals that may turn into a relation between a time interval and a time instant. On the other hand, there are scenarios where the type of topological is kept but the actual relation (e.g., before) is changed (e.g., to equal). An overview of the possible transitions between types of topological relations is given in Fig 1.

We start by discuss the transition expressed by the scenario one. Consider the granularities G and H such that $G \preccurlyeq H$ as defined in Fig. 2.

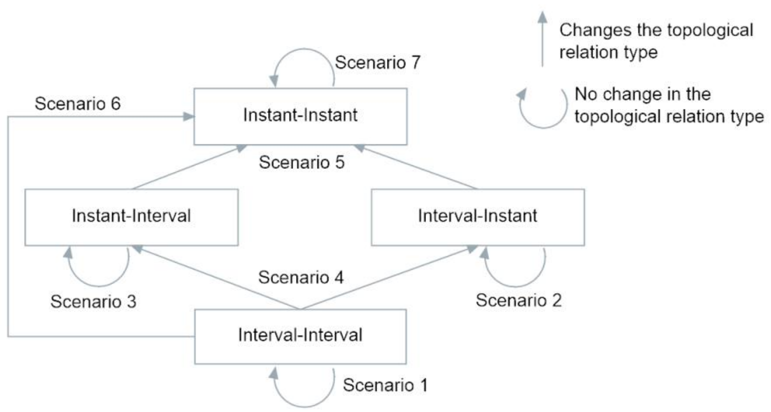

Fig. 1. Example of two granularities related by the finer-than relationship.

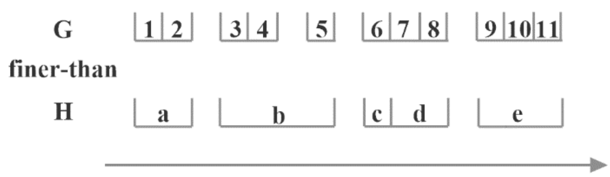

Fig. 2. Example of two granularities related by the finer-than relationship.

Let's consider $\alpha = Interval(\alpha^-, \alpha^+)$ and $\beta = Interval(\beta^-, \beta^+)$ be intervals of time defined over a granularity G; and, let $\alpha' = Interval(\alpha'^-, \alpha'^+)$ and $\beta' = Interval(\beta'^-, \beta'^+)$ be intervals of time, generalized from α and β respectively, of a granularity H. The relation between α' and β' may be the same relation verified by the intervals of time α and β, or may be changed due to the generalization.

In the first place, when α is equal to β in any generalization scenario α' will be equal to β'. By definition, $(\alpha^- = \beta^-) \wedge (\alpha^+ = \beta^+)$. Once the granularity G is finer than H then α^- and β^- will be contained by the same granule of H, and the same applies to the granules α^+ and β^+. Thus, in any generalization scenario α' will be equals to β'. The same reasoning can be applied to the meet relation. Imagine that, α meets β. We know a prior that $\alpha^+ = \beta^-$. Therefore, α^+ and β^- will be contained by the same granule of H. Consequently, in any generalization scenario, α' will meet β'.

Furthermore, a general rule can be stated regarding the following relations: before, overlaps, starts, during, and finishes. If the endpoints of α and β that are different before generalization remain different in α' and β', i.e., after the generalization, then the relation remain unchanged. Note that, G is finer than H. Thus, when two different instants of time of G are generalized for two different instants of H then the lesser complete relationship $<^c$ [3] between them are kept. Consequently, the relation between α and β will remain between α' and β'.

Nevertheless, there are some scenarios in which the relation between two intervals of time is changed due to the generalization of them. This issue is discussed below.

Suppose that, α occurs before β. If there is $h \in H$ such that $E(\alpha^+) \subseteq E(h) \wedge E(\beta^-) \subseteq E(h)$ then α' will meet β'. For example, $\alpha = Interval(1,3)$ and $\beta = Interval\ (5,6)$ such that α occurs before β. After the generalization we get: $\alpha' = Interval(a,b)$ meets $\beta' = Interval(b,c)$.

In case of α overlaps β then the relation between α' and β' can be changed to any relation apart from the before and during relation. Consider that there are two granules $x, y \in H$ such that $E(\alpha^-) \subseteq E(x) \wedge E(\beta^-) \subseteq E(x) \wedge E(\alpha^+) \subseteq E(y) \wedge E(\beta^+) \subseteq E(y)$. In this scenario, α' will be equals to β'. This can be illustrated by considering $\alpha = Interval(3,7)$ and $\beta = Interval\ (4,8)$. After the generalization we get: $\alpha' = Interval(b,d)$ equals $\beta' = Interval(b,d)$. However, if there is one granule $h \in H$ such that $E(\alpha^+) \subseteq E(h) \wedge E(\beta^-) \subseteq E(h)$ then α' will meet β'. For instance, if $\alpha = Interval(1,5)$ and $\beta = Interval\ (3,6)$ then $\alpha' = Interval(a,b)$ meets $\beta' = Interval(b,c)$. Now, if there are three granules $h \in H$ such that $E(\alpha^-) \subseteq E(h) \wedge E(\beta^-) \subseteq E(h)$ then α' will start β'. For example, $\alpha = Interval(3,7)$ overlaps $\beta = Interval\ (5,10)$ becomes $\alpha' = Interval(b,d)$ starts $\beta' = Interval\ (b,e)$. But if that granule exist such that $E(\alpha^+) \subseteq E(h) \wedge E(\beta^+) \subseteq E(h)$ then α' will be finished by β'. For example, $\alpha = Interval(6,10)$ overlaps $\beta = Interval\ (7,11)$ becomes $\alpha' = Interval(c,e)$ is finished by $\beta' = Interval\ (d,e)$.

Let's consider the scenario in which α starts β. If there is $h \in H$ such that $E(\alpha^+) \subseteq E(h) \wedge E(\beta^+) \subseteq E(h)$ then α' will become equal to β'.

Consider that α occurs during β. For this case, if there are two granules $x, y \in H$ such that $E(\alpha^-) \subseteq E(x) \wedge E(\beta^-) \subseteq E(x) \wedge E(\alpha^+) \subseteq E(y) \wedge E(\beta^+) \subseteq E(y)$ then α' will be equals to β'. Consider the following intervals: $\alpha = Interval(4,7)$ occurs during $\beta = Interval\ (3,8)$. After the generalization $\alpha' = Interval(b,d)$ is equal to $\beta' = Interval\ (b,d)$.

Nevertheless, if there are three granules $h \in H$ such that $E(\alpha^-) \subseteq E(h) \wedge E(\beta^-) \subseteq E(h)$ then α' will start β'. For example, $\alpha = Interval(4,6)$ during $\beta = Interval\ (3,8)$ becomes $\alpha' = Interval(b,c)$ starts $\beta' = Interval\ (b,d)$. Contrary, if that granule exist such that $E(\alpha^+) \subseteq E(h) \wedge E(\beta^+) \subseteq E(h)$ then α' will finish β'. For example, $\alpha = Interval(6,9)$ during $\beta = Interval\ (5, 10)$ becomes $\alpha' = Interval(c, e)$ finishes $\beta' = Interval\ (b, e)$.

Finally, let's assume α finishes β. In this case, if there is $h \in H$ such that $E(\alpha^-) \subseteq E(h) \wedge E(\beta^-) \subseteq E(h)$ then α' will become equal to β'. For example, $\alpha = Interval(2,5)$ finishes $\beta = Interval\ (1, 5)$ becomes $\alpha' = Interval(a, b)$ equal to $\beta' = Interval\ (a, b)$. An overview of the previous discussion is given in Table 1.

Table 1. – Possible transitions in the **scenario 1**.

	Before	Equals	Overlaps	Meets	Starts	During	Finishes
Before	✓	×	×	✓	×	×	×
Equals	×	✓	×	×	×	×	×
Overlaps	×	✓	✓	✓	✓	×	✓*
Meets	×	×	×	✓	×	×	×
Starts	×	✓	×	×	✓	×	×
During	×	✓	×	×	✓	✓	✓
Finishes	×	✓	×	×	×	×	✓

So far it was assumed that the generalization of any interval of time of G results into an interval of time of H. However, the generalization of an interval of time of G may result into an instant of time of H which changes a relation between two intervals of time to a relation between an instant and an interval of time or the other way around (scenario two).

Let's consider again $\alpha = Interval(\alpha^-, \alpha^+)$ and $\beta = Interval(\beta^-, \beta^+)$ be intervals of time defined over a granularity G. There are two ways of a relation between α and β to become a relation between an instant and an interval of time. The first one consists in α turn out to be an instant α' of H and β remains an interval of time $\beta' = Interval(\beta'^-, \beta'^+)$ of H.

In these contexts, whenever there is an granule $h \in H$ such that $E(\beta^-) \subseteq E(h) \wedge E(\alpha) \subseteq E(h)$ then any relation (except finish relation) between α and β will become α' starts β'. For example, $\alpha = Interval(3,4)$ occurs before $\beta = Interval\ (5, 8)$ becomes $\alpha' = b$ starts $\beta' = Interval\ (b, d)$. Another example can be: $\alpha = Interval(3,5)$ overlaps $\beta = Interval\ (4, 7)$ becomes $\alpha' = b$ starts $\beta' = Interval\ (b, d)$.

Suppose that, α occurs before β. If there is $h \in H$ such that $E(\beta^-) \subseteq E(h) \wedge \alpha' \neq h$ then α' will occur before β'. For example, $\alpha = Interval(1,2)$ occurs before $\beta = Interval\ (3, 6)$. After the generalization we get: $\alpha' = a$ occurs before $\beta' = Interval(b, c)$.

Consider that, α occurs during β. If there is $x, y \in H$ such that $E(\beta^-) \subseteq E(x) \wedge E(\beta^+) \subseteq E(y) \wedge x \neq \alpha' \neq y$ then α' will occur during β'. For example, $\alpha = Interval(2,5)$ occurs during $\beta = Interval\ (2, 6)$ becomes $\alpha' = b$ occurs during $\beta' = Interval\ (a, c)$. Contrary, if there is $h \in H$ such that $E(\beta^+) \subseteq E(h) \wedge \alpha' = h$ then α' will finish β'. For example, $\alpha = Interval(9,10)$ occurs during $\beta = Interval\ (7, 11)$ becomes $\alpha' = e$ finishes $\beta' = Interval\ (d, e)$.

Let's assume α finishes β. In this case, if there is $h \in H$ such that $E(\beta^{+}) \subseteq E(h) \wedge \alpha' = h$ then α' and β' will keep the relation. For example, $\alpha = Interval(4{,}5)$ finishes $\beta = Interval\ (1, 5)$ then $\alpha' = b$ also finishes $\beta' = Interval\ (a, b)$. Finally, if two intervals of time are equal then this discussion is not applicable because there is no scenario in which just one of them becomes an instant. Either α and β remain equal as intervals of time or as instants of time. An overview of the previous discussion is given in Table 2.

Table 2. – Possible transitions in the **scenario 4.**

		Instant – Interval Relation				
		Before	Starts	During	Finishes	After
Interval – Interval Relation	Before	✓	✓	×	×	×
	Equals	Not Applicable				
	Overlaps	×	✓	×	×	×
	Meets	×	✓	×	×	×
	Starts	×	✓	×	×	×
	During	×	✓	✓	✓	×
	Finishes	×	×	×	✓	×

The other possible scenario consists in α remains an interval of time $\alpha' = Interval(\alpha'^{-}, \alpha'^{+})$ of H and β turns out to be an instant of time β' of H. In this case and whenever there is an granule $h \in H$ such that $E(\alpha^{+}) \subseteq E(h)\ \wedge \beta'\ = h$ then a before, overlaps or meets relation between α and β will become α' finished by β'. For example, $\alpha = Interval(1{,}3)$ occurs before $\beta = Interval\ (4, 5)$ becomes $\alpha' = Interval\ (a, b)$ finished by $\beta' = b$.

Suppose that, α occurs before β. If there is $h \in H$ such that $E(\alpha^{+}) \subseteq E(h)\ \wedge \beta'\ \neq h$ then α' will occur before β'. For example, $\alpha = Interval(1{,}3)$ occurs before $\beta = Interval\ (7, 8)$. After generalization, $\alpha' = Interval\ (a, b)$ occurs before $\beta' = d$. Regarding the relation equals, starts, during and finishes this discussion is not applicable. In these cases and by the relation definition, the extent of β contains the extent of α. In order to β turns out to be an instant β' implies that α becomes also an instant α'. An overview of the previous discussion is given in Table 3.

Table 3. – Possible transitions in the **scenario 4.**

		Interval – Instant Relation				
		Before	Starts	During	Finishes	After
Interval – Interval Relation	Before	✓	×	×	✓*	×
	Equals	Not Applicable				
	Overlaps	×	×	×	✓*	×
	Meets	×	×	×	✓*	×
	Starts	Not Applicable				
	During	Not Applicable				
	Finishes	Not Applicable				

Furthermore, the generalization can turn a relation between intervals of time into a relation between instants of time (**scenario 6**). Let's assume α' and β' are two instants of time of H that result from the generalization of α and β, respectively. In these circumstances, we can conclude that α' and β' will be equal in any generalization scenario except if α and β are related through the before relation. Note that, these circumstances the extent of β intersects the extent of α. As a result, in order to α and β turn out to be instants implies that α' and β' are equal.

When α occurs before β, after the generalization, α' and β' can also be equal or the before relation is "maintained". If there is $h \in H$ such that $E(\alpha^-) \subseteq E(h) \wedge E(\beta^+) \subseteq E(h)$ then α' will occur before β'.

Until now, the discussion about the generalization of temporal terms and temporal relations has its starting point from the generalization of two intervals of time. Now, let's consider α and $\beta = Interval(\beta^-, \beta^+)$ be an instant and an interval of time defined over a granularity G, correspondingly; and, let α' and $\beta' = Interval(\beta'^-, \beta'^+)$ be an instant and an interval of time, generalized from α and β respectively, of a granularity H (**scenario 4**).

A general rule can be stated regarding the relations between an instant and an interval of time: if the granules involved (α and the endpoints of β) that are different before generalization remain different in α' and β', i.e., after the generalization, then the relation remain unchanged. The rationale is the same as it was in the generalization between intervals of time. This is also applicable in case of interval-instant relations.

There are a few scenarios in which the relation between α and β is different from the relation between α' and β'. Suppose that, α occurs before β. If there is $h \in H$ such that $E(\alpha) \subseteq E(h) \wedge E(\beta^-) \subseteq E(h)$ then α' will occur before β'. Now, consider that α occurs during β. If there is $h \in H$ such that $E(\alpha) \subseteq E(h) \wedge E(\beta^-) \subseteq E(h)$ then α' will starts β'. Contrary, if there is $h \in H$ such that $E(\alpha) \subseteq E(h) \wedge E(\beta^+) \subseteq E(h)$ then α' will finish β'. On the other hand, let's consider α occurs after β. If there is $h \in H$ such that $E(\alpha) \subseteq E(h) \wedge E(\beta^+) \subseteq E(h)$ then α' will be finished by β'.An overview of the previous discussion is given in Table 4.

A similar discussion can be made if we consider α as an interval of time and β an instant of time (**scenario 3**). An overview of the possible transitions is displayed in Table 5.

Table 4. – Possible transitions in the **scenario 3.**

		Instant – Interval Relation				
		Before	Starts	During	Finishes	After
Instant – Interval Relation	Before	✓	✓	×	×	×
	Starts	×	✓	×	×	×
	During	×	✓	✓	✓	×
	Finishes	×	×	×	✓	×
	After	×	×	×	✓*	✓

Table 5. – Possible transitions in the **scenario 2.**

		Interval – Instant Relation				
		Before	Starts^{-1}	During^{-1}	Finishes^{-1}	After
Interval – Instant Relation	Before	✓	×	×	✓*	×
	Start^{-1}	×	✓	×	×	×
	During^{-1}	×	✓	✓	✓	×
	Finishes^{-1}	×	×	×	✓	×
	After	×	×	×	✓*	✓

In same the way, the generalization can turn a relation between intervals of time into a relation between instants of time also a relation between an instant and an interval of time (or vice-versa) can become a relation between two instants of time (**scenario 5**). In case of α and β are related through the start, during or finishes relation, in any generalization scenario α' and β' will be equal. The reason is similar to the one exposed in the case of intervals of time. If α occurs before or after β then α' may keep occur before, or after respectively β', or be equal. The circumstances in which these changes occurs are similar to the scenario four.

Last but not least, when two different instants of time of G, α and β, are generalized for two instants of H, α' and β' (**scenario 7**), the relationship between α and β are kept if and only if $E(\alpha') \cap E(\beta') = \emptyset$. Otherwise both instants of time become the same (at the granularity H). Furthermore, if two instants of time of G are equal, after the generalization, they remain equal.